

Multi-Variable Calculus Past Paper Question Pack

MEI FP3 June 2006

A surface has equation $x^2 - 4xy + 3y^2 - 2z^2 - 63 = 0$.

- (i) Find a normal vector at the point (x, y, z) on the surface. [4]
 - (ii) Find the equation of the tangent plane to the surface at the point $Q(17, 4, 1)$. [4]
 - (iv) Show that there is no point on the surface where the normal line is parallel to the z -axis. [4]
 - (v) Find the two values of k for which $5x - 6y + 2z = k$ is a tangent plane to the surface. [8]
- (iii) removed

A surface has equation $z = xy^2 - 4x^2y - 2x^3 + 27x^2 - 36x + 20$.

- (i) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. [3]
- (ii) Find the coordinates of the four stationary points on the surface, showing that one of them is $(2, 4, 8)$. [8]
- (iii) Sketch, on separate diagrams, the sections of the surface defined by $x = 2$ and by $y = 4$. Indicate the point $(2, 4, 8)$ on these sections, and deduce that it is neither a maximum nor a minimum. [6]
- (iv) Show that there are just two points on the surface where the normal line is parallel to the vector $36\mathbf{i} + \mathbf{k}$, and find the coordinates of these points. [7]

MEI FP3 June 2008

You are given $g(x, y, z) = 6xz - (x + 2y + 3z)^2$.

(i) Find $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$ and $\frac{\partial g}{\partial z}$. [4]

A surface S has equation $g(x, y, z) = 125$.

(ii) Find the equation of the normal line to S at the point $P(7, -7.5, 3)$. [3]

(iv) Show that there is no point on S at which the normal line is parallel to the z -axis. [4]

(v) Find the two points on S at which the tangent plane is parallel to $x + 5y = 0$. [8]

(iii) removed

A surface has equation $z = 3x(x + y)^3 - 2x^3 + 24x$.

(i) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. [4]

(ii) Find the coordinates of the three stationary points on the surface. [7]

(iii) Find the equation of the normal line at the point P (1, -2, 19) on the surface. [3]

(v) Show that there is only one point on the surface at which the tangent plane has an equation of the form $27x - z = d$. Find the coordinates of this point and the corresponding value of d . [6]

(iv) removed

MEI FP3 June 2011

A surface S has equation $z = 8y^3 - 6x^2y - 15x^2 + 36x$.

- (i) Sketch the section of S given by $y = -3$, and sketch the section of S given by $x = -6$. Your sketches should include the coordinates of any stationary points but need not include the coordinates of the points where the sections cross the axes. [7]
- (ii) From your sketches in part (i), deduce that $(-6, -3, -324)$ is a stationary point on S , and state the nature of this stationary point. [2]
- (iii) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, and hence find the coordinates of the other three stationary points on S . [8]
- (iv) Show that there are exactly two values of k for which the plane with equation

$$120x - z = k$$

is a tangent plane to S , and find these values of k . [7]

MEI FP3 June 2012

You are given that $g(x, y, z) = x^2 + 2y^2 - z^2 + 2xz + 2yz + 4z - 3$.

- (i) Find $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$ and $\frac{\partial g}{\partial z}$. [3]

The surface S has equation $g(x, y, z) = 0$, and $P(-2, -1, 1)$ is a point on S .

- (ii) Find an equation for the normal line to the surface S at the point P . [3]

- (iv) Show that there is no point on S at which the normal line is parallel to the z -axis. [5]

- (v) Given that $x + y + z = k$ is a tangent plane to the surface S , find the two possible values of k . [8]

(Part (iii) removed deliberately)

MEI FP3 June 2013

A surface has equation $z = 2(x^3 + y^3) + 3(x^2 + y^2) + 12xy$.

- (i) For a point on the surface at which $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$, show that either $y = x$ or $y = 1 - x$. [5]
- (ii) Show that there are exactly two stationary points on the surface, and find their coordinates. [7]
- (iv) Find the four points on the surface at which the normal line is parallel to the vector $24\mathbf{i} + 24\mathbf{j} - \mathbf{k}$. [7]

MEI FP3 June 2014

A surface S has equation $g(x, y, z) = 0$, where $g(x, y, z) = x^2 + 3y^2 + 2z^2 + 2yz + 6xz - 4xy - 24$.
 $P(2, 6, -2)$ is a point on the surface S .

(i) Find $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$ and $\frac{\partial g}{\partial z}$. [3]

(ii) Find the equation of the normal line to the surface S at the point P . [3]

(iv) Find the coordinates of the two points on the surface at which the normal line is parallel to the y -axis. [6]

(v) Given that $10x - y + 2z = 6$ is the equation of a tangent plane to the surface S , find the coordinates of the point of contact. [8]

(Part (iii) removed deliberately)